

## □ 22 □ □□□□□□□

1□□□□□  $f(x)=(x-2)e^x-a(x-1)^2$  □  $a\in R$  □

□1□□□  $f(x)$  □□□□□

□2□□  $f(x)$  □□□□□□□  $a$ □□□□□□

□□□□□□□1□□  $f(x)=(x-2)e^x-a(x-1)^2$  □

□□  $f(x)=(x-1)e^x-2a(x-1)=(x-1)(e^x-2a)$  □

① □  $a, 0$  □□□  $f(x)>0$  □□□  $x>1$  □□  $f(x)<0$  □□□  $x<1$  □

□□  $f(x)$  □  $(-\infty,1)$  □□□□  $(1,+\infty)$  □□□

② □  $a>0$  □□□  $f(x)=0$  □□□  $x=1$  □  $x=\ln 2a$  □

□  $a=\frac{e}{2}$  □□  $f(x)=0$  □□□□□□  $f(x)$  □  $R$ □□□□

□  $0<a<\frac{e}{2}$  □□□  $f(x)>0$  □□□  $x>1$  □  $x<\ln(2a)$  □

□  $f(x)<0$  □□□  $\ln(2a)<x<1$  □

□□  $f(x)$  □  $(-\infty, \ln(2a))$  □  $(1,+\infty)$  □□□

□  $(\ln(2a), 1)$  □□□

□  $a>\frac{e}{2}$  □□  $f(x)>0$  □□□  $x<1$  □  $x>\ln(2a)$  □

□  $f(x)<0$  □□□  $1<x<\ln(2a)$

□□  $f(x)$  □  $(-\infty,1)$  □  $(\ln(2a), +\infty)$  □□□□  $(1, \ln(2a))$  □□□

□□□□  $a, 0$  □□  $f(x)$  □  $(-\infty,1)$  □□□□  $(1,+\infty)$  □□□

$$\square a > 0 \square \square a = \frac{e}{2} \square \square f(x) \square R \square \square \square \square$$

$$0 < a < \frac{e}{2} \square \square f(x) \square (-\infty, \ln(2a)) \square (1, +\infty) \square \square \square \square (\ln(2a) \square 1) \square \square \square$$

$$a > \frac{e}{2} \square \square f(x) \square (-\infty, 1) \square (\ln(2a) \square +\infty) \square \square \square \square (1 \square \ln(2a)) \square \square \square$$

$$\square 2 \square \textcircled{1} \square \square 1 \square \square \square \square \square a < 0 \square \square f(x) \square (-\infty, 1) \square \square \square \square (1, +\infty) \square \square \square$$

$$\square f \square 1 \square = -e < 0 \square f \square 2 \square = -a > 0 \square \square f(x) \square (1, 2) \square \square \square 1 \square \square \square \square$$

$$\square b \square \square b < 0 \square \square b < \ln(-\frac{a}{2}) \square$$

$$\square f \square b \square = (b-2)e^b - a(b-1)^2 > -\frac{a}{2}(b-2) - a(b-1)^2 = -a(b-\frac{3}{2}) > 0 \square$$

$$\square f(x) \square (b, 1) \square \square \square \square 1 \square \square \square \square$$

$$\square a < 0 \square \square f(x) \square 2 \square \square \square \square$$

$$\textcircled{2} \square a = 0 \square \square f(x) = (x-2)e^x \square \square \square f(x) \square \square \square \square \square x = 2 \square \square \square \square \square \square$$

$$\textcircled{3} \square a > 0 \square \square \square a = \frac{e}{2} \square \square f(x) \square R \square \square \square f(x) \square \square \square 2 \square \square \square \square \square \square \square \square \square$$

$$\square 0 < a < \frac{e}{2} \square \square f(x) \square (1, +\infty) \square \square \square \square x, 1 \square \square f(x) < 0 \square f(x) \square \square \square 2 \square \square \square \square \square \square \square \square \square$$

$$\square a > \frac{e}{2} \square \square f(x) \square (-\infty, 1) \square \square \square \square (1 \square \ln(2a)) \square \square \square (\ln(2a) \square +\infty) \square \square \square$$

$$f(x) \square \square \square = f \square 1 \square = -e < 0 \square \square f(x) \square \square \square 2 \square \square \square \square \square \square \square \square \square$$

$$\square \square \square f(x) \square \square \square \square \square \square a \square \square \square \square \square (-\infty, 0) \square$$

$$2 \square \square \square \square \square f(x) = \ln x - \frac{1}{2}ax^2 \square$$

$$\square 1 \square \square \square f(x) \square \square \square \square \square \square$$

$$\square 2 \square \square f(x) \square \square \square \square \square \square \square a \square \square \square \square \square \square \square$$

$$\text{f(0)=0} \quad f(x) \text{ in } (0, +\infty) \quad f(x) = \frac{1-a x^2}{x}$$

$$a, 0 \quad f(x) > 0 \quad f(x) \text{ in } (0, +\infty)$$

$$a > 0 \quad f(x) > 0 \quad 0 < x < \frac{\sqrt{a}}{a} \quad f(x) < 0 \quad x > \frac{\sqrt{a}}{a} \quad f(x) \text{ in } (0, \frac{\sqrt{a}}{a}) \quad (\frac{\sqrt{a}}{a}, +\infty)$$

$$a, 0 \quad f(x) \text{ in } (0, +\infty)$$

$$a > 0 \quad f(x) \text{ in } (0, \frac{\sqrt{a}}{a}) \quad (\frac{\sqrt{a}}{a}, +\infty)$$

$$2 \text{ in } (0, +\infty) \quad f(x) \text{ in } (0, +\infty) \quad f(x)$$

$$a > 0 \quad f(x) \text{ in } (0, \frac{\sqrt{a}}{a}) \quad (\frac{\sqrt{a}}{a}, +\infty) \quad f(x)_{\max} = f(\frac{\sqrt{a}}{a}) = n \frac{1}{\sqrt{a}} - \frac{1}{2} \cdot a \cdot (\frac{1}{\sqrt{a}})^2 = -\frac{1}{2} \ln(a+1)$$

$$a \cdot \frac{1}{e} \quad f(x)_{\max} = f(\frac{\sqrt{a}}{a}) = -\frac{1}{2} \ln(a+1), 0 \quad f(x)$$

$$0 < a < \frac{1}{e} \quad f(x)_{\max} = f(\frac{\sqrt{a}}{a}) = -\frac{1}{2} \ln(a+1) > 0$$

$$1 \in (0, \frac{1}{\sqrt{a}}) \quad f(1) = n \cdot \frac{1}{2} \cdot a \cdot 1^2 = -\frac{1}{2} a < 0$$

$$f(x) \text{ in } (0, \frac{1}{\sqrt{a}})$$

$$\frac{2}{a} > \frac{1}{\sqrt{a}} \quad f(\frac{2}{a}) = n \frac{2}{a} - \frac{1}{2} \cdot a \cdot (\frac{2}{a})^2 = n \frac{2}{a} - \frac{2}{a} < \frac{2}{a} - \frac{2}{a} = 0 \quad \ln x < x$$

$$f(x) \text{ in } (\frac{1}{\sqrt{a}}, +\infty)$$

$$a \text{ in } (0, \frac{1}{e})$$

$$3 \text{ 证明 } f(x) = e^x [e^x - 2(a+1)] + 2ax(e^{a+1} - a - 1)$$

$$1 \text{ 证明 } f(x) \text{ 在 } [0, 1] \text{ 上单调递增}$$

$$2 \text{ 证明 } f(x) \text{ 在 } [0, 1] \text{ 上单调递减}$$

$$3 \text{ 证明 } f(x) = 2(e^x - 1)(e^x - a)$$

$$4 \text{ 证明 } a, 0 < e^x - a < 0$$

$$5 \text{ 证明 } f(x) < 0 \text{ 在 } (-\infty, 0) \text{ 上成立}$$

$$6 \text{ 证明 } f(x) > 0 \text{ 在 } (0, +\infty) \text{ 上成立}$$

$$7 \text{ 证明 } a > 0 \text{ 时 } f(x) = 0 \text{ 的根 } x_1 = \ln a, x_2 = 0$$

$$8 \text{ 证明 } a = 1 \text{ 时 } f(x) \text{ 在 } [0, 1] \text{ 上单调递增}$$

$$9 \text{ 证明 } 0 < a < 1 \text{ 时 } f(x) \text{ 在 } [0, 1] \text{ 上单调递减}$$

$$10 \text{ 证明 } x < \ln a \text{ 时 } f(x) > 0 \text{ 在 } \ln a < x < 0 \text{ 时 } f(x) < 0$$

$$11 \text{ 证明 } f(x) \text{ 在 } (-\infty, \ln a) \text{ 上单调递增, 在 } (\ln a, 0) \text{ 上单调递减}$$

$$12 \text{ 证明 } a, 0 \text{ 时 } f(x) \text{ 在 } (-\infty, 0) \text{ 上单调递增, 在 } (0, +\infty) \text{ 上单调递减}$$

$$13 \text{ 证明 } 0 < a < 1 \text{ 时 } f(x) \text{ 在 } (-\infty, \ln a) \text{ 上单调递减, 在 } (\ln a, 0) \text{ 上单调递增}$$

$$14 \text{ 证明 } a = 1 \text{ 时 } f(x) \text{ 在 } [0, 1] \text{ 上单调递增}$$

$$15 \text{ 证明 } a = 1 \text{ 时 } f(x) \text{ 在 } [0, 1] \text{ 上单调递减}$$

$$16 \text{ 证明 } 0 < a < 1 \text{ 时 } f(x) \text{ 在 } (-\infty, \ln a) \text{ 上单调递增, 在 } (\ln a, 0) \text{ 上单调递减}$$

$$17 \text{ 证明 } x = \ln a \text{ 时 } f(x) \text{ 在 } [0, 1] \text{ 上单调递增}$$

□□□  $f(x)$  □□□□ 2 □□□□

③ □  $a=0$  □□  $f(x) = e^x(e^x - 2)$  □□  $f(x) = 0$  □  $x = \ln 2$  □

□□□  $f(x)$  □□ 1 □□□□

④ □  $a < 0$  □□  $f(x)$  □  $(-\infty, 0)$  □□□□  $(0, +\infty)$  □□□□

□  $f(x)_{\min} = f(0) = -1 - 2a$  □

□  $f(x)$  □ 2 □□□□ □  $f(0) < 0$  □

□□□  $a > -\frac{1}{2}$  □  $\therefore -\frac{1}{2} < a < 0$  □

□  $f(1)$  □  $= [e^b - 2(a+1)] + 2a > 0$  □

□  $b < \frac{(a+1)^2}{2a}$  □□  $f(b)$  □  $= [e^b - (a+1)]^2 + 2ab > [e^b - (a+1)]^2 \dots 0$  □

□  $f(x)$  □  $(-\infty, 0)$  □  $(0, +\infty)$  □□ 1 □□□□

□□□  $a$  □□□□□□□  $(-\frac{1}{2}, 0)$  □

4□□□□□  $f(x) = xe^x + a(x+1)^2 (a \in \mathbb{R})$  □

□1□□□  $f(x)$  □□□□□

□2□□  $f(x)$  □□□□□□□  $a$  □□□□□□□

□□□□□□□1□□  $f(x) = xe^x + a(x+1)^2$  □

□□  $f(x) = (x+1)e^x + 2a(x+1) = (x+1)(e^x + 2a)$  □

① □  $a > 0$  □□□  $f(x) > 0$  □□□  $x > -1$  □□  $f(x) < 0$  □□□  $x < -1$  □

$$f(x) \text{ } (-\infty, -1) \text{ } (-1, +\infty)$$

$$\textcircled{2} \text{ } a < 0 \text{ } f(x) = 0 \text{ } x = -1 \text{ } x = \ln(-2a)$$

$$a < -\frac{1}{2e} \text{ } f(x) = (x+1)(e^x - e^{-1}) \text{ } x, -1 \text{ } f(x) > 0 \text{ } x > -1 \text{ } f(x) > 0$$

$$\therefore \forall x \in \mathbb{R} \text{ } f(x) > 0 \text{ } f(x) \text{ } \mathbb{R}$$

$$a < -\frac{1}{2e} \text{ } \ln(-2a) > -1 \text{ } f(x) > 0 \text{ } x < -1 \text{ } x > \ln(-2a)$$

$$f(x) < 0 \text{ } 1 < x < \ln(-2a)$$

$$f(x) \text{ } (-\infty, -1) \text{ } (\ln(-2a), +\infty)$$

$$(-1, \ln(-2a))$$

$$0 > a > -\frac{1}{2e} \text{ } \ln(-2a) < -1 \text{ } f(x) > 0 \text{ } x < \ln(-2a) \text{ } x > -1$$

$$f(x) < 0 \text{ } \ln(-2a) < x < -1$$

$$f(x) \text{ } (-\infty, \ln(-2a)) \text{ } (-1, +\infty) \text{ } (\ln(-2a), -1)$$

$$\textcircled{2} \textcircled{1} \text{ } a > 0 \text{ } f(x) \text{ } (-\infty, -1) \text{ } (-1, +\infty)$$

$$f(-1) = -\frac{1}{e} \text{ } f(0) = a \text{ } b \text{ } b < -1 \text{ } b-2 < \ln \frac{a}{2} \text{ } f(b-2) > \frac{a}{2}(b-2) + a(b-1)^2 = a(b - \frac{3}{2}b) > 0$$

$$\therefore f(x) \text{ } \mathbb{R}$$

$$\textcircled{2} \text{ } a = 0 \text{ } f(x) = xe^x \text{ } f(x) \text{ } x = 0$$

$$\textcircled{3} \text{ } a < 0$$

$$a < -\frac{1}{2e} \text{ } f(x) \text{ } (-1, \ln(-2a))$$

$$(-\infty, -1) \text{ } (\ln(-2a), +\infty)$$

Em  $x_0 - 1$  tem  $f(x) < 0$  e  $f(x)$  é crescente

Logo  $\frac{1}{2e} \leq f(x) < 0$  em  $(-1, +\infty)$  e  $x_0 - 1$  tem  $f(x) < 0$  e  $f(x)$  é crescente  
 e  $f(x)$  é crescente em  $a$  em  $(0, +\infty)$

$$5 \text{ tem } f(x) = ae^{2x} + (a-2)e^x - x$$

1 tem  $f(x)$  é crescente

2 tem  $f(x)$  é crescente em  $a$  é crescente

$$\text{Logo } 1 \text{ tem } f(x) = ae^{2x} + (a-2)e^x - x \text{ e } f(x) = 2ae^{2x} + (a-2)e^x - 1$$

$$\text{e } e^x > 0 \text{ e } e^x > 0$$

$$\therefore \text{em } a, 0 \text{ tem } f(x) < 0$$

$$\therefore f(x) \text{ é } R \text{ é crescente}$$

$$\text{em } a > 0 \text{ tem } f(x) = (2e^x + 1)(ae^x - 1) = 2a(e^x + \frac{1}{2})(e^x - \frac{1}{a})$$

$$\text{em } f(x) = 0 \text{ tem } x = \ln \frac{1}{a}$$

$$\text{em } f(x) > 0 \text{ tem } x > \ln \frac{1}{a}$$

$$\text{em } f(x) < 0 \text{ tem } x < \ln \frac{1}{a}$$

$$\therefore x \in (-\infty, \ln \frac{1}{a}) \text{ tem } f(x) \text{ é crescente e } x \in (\ln \frac{1}{a}, +\infty) \text{ é crescente}$$

$$\text{Logo em } a, 0 \text{ tem } f(x) \text{ é } R \text{ é crescente}$$

$$\text{em } a > 0 \text{ tem } f(x) \text{ é } (-\infty, \ln \frac{1}{a}) \text{ é crescente e } (\ln \frac{1}{a}, +\infty) \text{ é crescente}$$

$$\text{2 tem } a, 0 \text{ tem } 1 \text{ tem } f(x) \text{ é crescente}$$

$$\square a > 0 \square \square f(x) = ae^{2x} + (a-2)e^x - x \square$$

$$\square X \rightarrow -\infty \square \square e^{2x} \rightarrow 0 \square e^x \rightarrow 0 \square$$

$$\therefore \square X \rightarrow -\infty \square \square f(x) \rightarrow +\infty \square$$

$$\square X \rightarrow \infty \square \square e^{2x} \rightarrow +\infty \square \square \square \square \square \square e^x \square X \square$$

$$\therefore \square X \rightarrow \infty \square \square f(x) \rightarrow +\infty \square$$

$$\therefore \square \square \square \square \square \square \square f(x) \square \square \square \square \square \square 0 \square \square \square$$

$$\square f(x) \square \left(-\infty, \ln \frac{1}{a}\right) \square \square \square \square \square \square \left(\ln \frac{1}{a}, +\infty\right) \square \square \square \square \square$$

$$\therefore f(x)_{\min} = f\left(\ln \frac{1}{a}\right) = a \times \left(\frac{1}{a}\right) + (a-2) \times \frac{1}{a} - \ln \frac{1}{a} < 0 \square$$

$$\therefore 1 - \frac{1}{a} - \ln \frac{1}{a} < 0 \square \square \ln \frac{1}{a} + \frac{1}{a} - 1 > 0 \square$$

$$\square t = \frac{1}{a} \square \square g(t) = \ln t + t - 1 \square (t > 0) \square$$

$$\square \square g'(t) = \frac{1}{t} + 1 \square \square g'(1) = 0 \square$$

$$\therefore t = \frac{1}{a} > 1 \square \square \square \square 0 < a < 1 \square$$

$$\therefore a \square \square \square \square (0, 1) \square$$

$$\square \square \square \square \square 1 \square \square f(x) = ae^{2x} + (a-2)e^x - x \square \square f'(x) = 2ae^{2x} + (a-2)e^x - 1 \square$$

$$\square e^{2x} > 0 \square e^x > 0$$

$$\therefore \square a, 0 \square \square f'(x) < 0 \square$$

$$\therefore f(x) \square R \square \square \square \square \square \square$$

$$\square a > 0 \square \square f(x) = (2e^x + 1)(ae^x - 1) = 2a\left(e^x + \frac{1}{2}\right)\left(e^x - \frac{1}{a}\right) \square$$



$$f'(x) = 0 \iff x = -\ln a$$

$$f'(x) > 0 \iff x > -\ln a$$

$$f'(x) < 0 \iff x < -\ln a$$

$$\therefore x \in (-\infty, -\ln a) \implies f'(x) < 0 \implies x \in (-\ln a, +\infty) \implies f'(x) > 0$$

$$\text{Hence } a, 0 < f(x) \in \mathbb{R}$$

$$a > 0 \implies f(x) \in (-\infty, -\ln a) \cup (-\ln a, +\infty)$$

$$2 \text{ (i) } a, 0 < f(x) \implies f(x) \in \mathbb{R}$$

$$\text{(ii) } a > 0 \implies f(x) \in \mathbb{R} \implies f(x)_{\min} = f(-\ln a) = 1 - \frac{1}{a} - \ln \frac{1}{a}$$

$$a = 1 \implies f(-\ln a) = 0 \implies f(x) \in \mathbb{R}$$

$$a \in (1, +\infty) \implies 1 - \frac{1}{a} - \ln \frac{1}{a} > 0 \implies f(-\ln a) > 0$$

$$f(x) \in \mathbb{R}$$

$$a \in (0, 1) \implies 1 - \frac{1}{a} - \ln \frac{1}{a} < 0 \implies f(-\ln a) < 0$$

$$f(-2) = ae^4 + (a-2)e^2 + 2 > -2e^2 + 2 > 0$$

$$f(x) \in (-\infty, -\ln a) \implies f(x) \in \mathbb{R}$$

$$\text{Hence } n_0 > \ln \left( \frac{3}{a} - 1 \right) \implies f(n_0) = e^{n_0} (ae^{n_0} + a - 2) - n_0 > e^{n_0} - n_0 > 2^{n_0} - n_0 > 0$$

$$\ln \left( \frac{3}{a} - 1 \right) > -\ln a$$

$$\implies (-\ln a, +\infty) \implies f(x) \in \mathbb{R}$$

$$\therefore a \in (0, 1) \implies f(x) \in \mathbb{R}$$

6. 设函数  $f(x) = ax^2 + (a-2)x - \ln x$

1. 求  $f(x)$  的极值

2. 若  $f(x) \geq a$  恒成立，求  $a$  的取值范围

解：1. 由  $f(x) = ax^2 + (a-2)x - \ln x$  ( $a \in \mathbb{R}$ )

$\therefore f'(x) = 2ax + (a-2) - \frac{1}{x} = \frac{2ax^2 + (a-2)x - 1}{x} = \frac{(2x+1)(ax-1)}{x}$  ( $x > 0$ )

当  $a \leq 0$  时， $f'(x) < 0$ ， $f(x)$  在  $(0, +\infty)$  上单调递减

当  $a > 0$  时， $f'(x) = 0$  在  $(0, \frac{1}{a})$  上无解，在  $(\frac{1}{a}, +\infty)$  上无解

当  $a > 0$  时， $f(x)$  在  $(0, \frac{1}{a})$  上单调递减，在  $(\frac{1}{a}, +\infty)$  上单调递增

2. 由 1 知，当  $a \leq 0$  时， $f(x)$  在  $(0, +\infty)$  上单调递减，故  $f(x) < f(1) = a - 1$

当  $a > 0$  时， $f(x)_{\min} = f(\frac{1}{a}) = a(\frac{1}{a})^2 + (a-2)\frac{1}{a} - \ln \frac{1}{a} = \frac{1}{a} + 1 + \ln a$

当  $x \rightarrow 0^+$  时， $f(x) > 0$

当  $x \rightarrow +\infty$  时， $f(x) > 0$

$\therefore f(\frac{1}{a}) = 1 + \ln a - \frac{1}{a} < 0$ ， $g(a) = 1 + \ln a - \frac{1}{a}$

$g'(a) = \frac{1}{a} + \frac{1}{a^2}$

$\therefore g'(a) > 0$

□  $g_{aa}(0, +\infty)$  □□□□□□

□  $g_{11} = 0$  □□□  $a < 1$  □

∴ □  $0 < a < 1$  □□□□  $f(x)$  □□□□□□□□□□ □12 □□

□□□□□□□□□□□□□□□□

7□□□□□  $f(x) = \frac{1}{2}e^{2x} - (a+1)e^x + ax$  □

□1□□□  $f(x)$  □□□□□

□2□□  $f(x)$  □□□□□□□  $a$  □□□□□□

□□□□□□□1□  $f(x) = (e^x - 1)(e^x - a)$  □

(i) □  $a, 0$  □□  $x \in (-\infty, 0)$  □□  $f(x) < 0$  □  $f(x)$  □□□

$x \in (0, +\infty)$  □□  $f(x) > 0$  □  $f(x)$  □□□

□  $a > 0$  □□□  $f(x) = 0$  □□□□  $x_1 = 0$  □  $x_2 = \ln a$  □

(ii) □  $a = 1$  □  $x_1 = x_2$  □  $f(x) \dots 0$  □□□□  $f(x)$  □  $R$  □□□

(iii) □  $0 < a < 1$  □  $x_1 > x_2$  □

□  $x \in (-\infty, \ln a)$  □□  $f(x) > 0$  □  $f(x)$  □□□

□  $x \in (\ln a, 0)$  □□  $f(x) < 0$  □  $f(x)$  □□□

□  $a \in (0, +\infty)$  □□  $f(x) > 0$  □  $f(x)$  □□□

(iv) □  $a > 1$  □  $x_1 < x_2$  □

$$\square \quad x \in (-\infty, 0) \quad \square \square \quad f(x) > 0 \quad \square \quad f(x) \quad \square \square \square$$

$$\square \quad x \in (0, \ln a) \quad \square \square \quad f(x) < 0 \quad \square \quad f(x) \quad \square \square \square$$

$$\square \quad x \in (\ln a, +\infty) \quad \square \square \quad f(x) > 0 \quad \square \quad f(x) \quad \square \square \square$$

$$\square \square \square \square \quad a, 0 \quad \square \quad f(x) \quad \square \quad (-\infty, 0) \quad \square \square \square \square \quad (0, +\infty) \quad \square \square \square$$

$$\square \quad a = 1 \quad \square \quad f(x) \quad \square \quad R \quad \square \square \square$$

$$\square \quad 0 < a < 1 \quad \square \quad f(x) \quad \square \quad (-\infty, \ln a) \quad \square \square \square \square \quad (\ln a, 0) \quad \square \square \square \square \quad (0, +\infty) \quad \square \square \square$$

$$\square \quad a > 1 \quad \square \quad f(x) \quad \square \quad (-\infty, 0) \quad \square \square \square \square \quad (0, \ln a) \quad \square \square \square \square \quad (\ln a, +\infty) \quad \square \square \square$$

$$\square \square \square \quad (i) \quad \square \quad a = 0 \quad \square \square \quad f(x) = \frac{1}{2} e^{2x} - e^x = e^x \left( \frac{1}{2} e^x - 1 \right) \quad \square$$

$$\square \quad f(x) = 0 \quad \square \square \square \square \quad x = \ln 2 \quad \square \square \square \quad 1 \quad \square \square \square \square \square \square \square \square$$

$$(ii) \quad \square \quad a < 0 \quad \square \square \square \square \quad 1 \quad \square \square \square \square$$

$$f(x) \quad \square \quad (-\infty, 0) \quad \square \square \square \square \quad (0, +\infty) \quad \square \square \square$$

$$\square \quad f(x) \quad \square \quad 2 \quad \square \square \square \square \square \square \quad f(0) = -a - \frac{1}{2} < 0 \quad \square \square \quad a > -\frac{1}{2} \quad \square$$

$$\square \quad f \quad \square \quad 1 \quad \square \quad = \frac{1}{2} e^2 - e + a(1 - e) > 0 \quad \square$$

$$\square \quad x \in (0, 1) \quad \square \square \quad f(x) > 0 \quad \square \square \square \square$$

$$\square \quad x < 0 \quad \square \square \quad f(x) > ax - (a+1)e^x > ax - (a+1) \quad \square$$

$$\square \quad x_0 < 1 + \frac{1}{a} < 0 \quad \square \square \quad f(x_0) > 0 \quad \square$$

$$\square \quad x \in (x_0, 0) \quad \square \square \quad f(x) > 0 \quad \square \square \square \square$$

$$\square \quad -\frac{1}{2} < a < 0 \quad \square \square \quad f(x) > 0 \quad \square \square \square \square \square \square \square \square$$

(iii)  $a=1$   $f(x)$   $R$   $2$

(iv)  $0 < a < 1$   $f(x)$   $(-\infty, \ln a)$   $(\ln a, 0)$   $(0, +\infty)$

$$f(\ln a) = \frac{1}{2} e^{2 \ln a} - (a+1) e^{\ln a} + a \ln a = \frac{1}{2} a^2 - a^2 - a + a \ln a = a \ln a - \frac{1}{2} (a-1)$$

$$\ln a - \frac{1}{2} (a-1) < 0 \quad f(\ln a) < 0$$

$f(x)$   $1$

(v)  $a > 1$   $f(x)$   $(-\infty, 0)$   $(0, \ln a)$   $(\ln a, +\infty)$

$$f(0) = \frac{1}{2} - a - 1 = -a - \frac{1}{2} < 0$$

$f(x)$   $1$

$f(x)$   $2$

$a$   $(-\frac{1}{2}, 0)$

$$f(x) = \frac{2x^2 - 1}{x} - a \ln x \quad (a \in R)$$

$f(x)$

$g(x) = e^x - \sin x$   $h(x) = g(x)(f(x) - 2x)$   $y = h(x)$   $a$

$$f(x) = 2 + \frac{1}{x^2} - \frac{a}{x} = \frac{2x^2 - ax + 1}{x^2} \quad x > 0 \quad \Delta = a^2 - 8$$

$$\Delta = a^2 - 8, 0 - 2\sqrt{2}, a, 2\sqrt{2} \quad f(x) \dots 0 \quad f(x) \quad (0, +\infty)$$

$$\Delta = a^2 - 8 > 0 \quad a > 2\sqrt{2} \quad a < -2\sqrt{2} \quad 2x^2 - ax + 1 = 0 \quad x_1 = \frac{a - \sqrt{a^2 - 8}}{4} \quad x_2 = \frac{a + \sqrt{a^2 - 8}}{4}$$

$$(i) \quad a > 2\sqrt{2} \quad x_1 = \frac{a - \sqrt{a^2 - 8}}{4} > 0 \quad x_2 = \frac{a + \sqrt{a^2 - 8}}{4} > 0$$

$$x \in (0, \frac{a - \sqrt{a^2 - 8}}{4}) \quad f'(x) > 0$$

$$x \in (\frac{a - \sqrt{a^2 - 8}}{4}, \frac{a + \sqrt{a^2 - 8}}{4}) \quad f'(x) < 0$$

$$x \in (\frac{a + \sqrt{a^2 - 8}}{4}, +\infty) \quad f'(x) > 0$$

$$(ii) \quad a < -2\sqrt{2} \quad x_1 = \frac{a - \sqrt{a^2 - 8}}{4} < 0 \quad x_2 = \frac{a + \sqrt{a^2 - 8}}{4} < 0$$

$$x \in (0, +\infty) \quad f'(x) > 0$$

$$g(x) = e^x - \sin x \quad g'(x) = e^x - \cos x$$

$$x > 0 \quad e^x > 1 \quad \cos x, 1 \quad g'(x) = e^x - \cos x > 0 \quad g(x) \quad (0, +\infty) \quad g(0) = 1 > 0$$

$$g(x) \quad (0, +\infty)$$

$$h(x) = g(x)(f(x) - 2x) = -g(x)(\frac{1}{x} + a \ln x)$$

$$F(x) = \frac{1}{x} + a \ln x \quad x > 0$$

$$F(x) = \frac{ax - 1}{x^2} \quad x > 0$$

$$a, 0 \quad F(x) < 0 \quad F(x) \quad (0, +\infty) \quad 1$$

$$a > 0 \quad F(x) \quad (0, \frac{1}{a}) \quad (\frac{1}{a}, +\infty)$$

$$x \rightarrow 0 \quad F(x) \rightarrow -\infty \quad x \rightarrow +\infty \quad F(x) \rightarrow +\infty$$

$$F(\frac{1}{a}) = a - a \ln a < 0$$

$$a > e$$

$$a \in (e, +\infty)$$

$$f(x) = ae^{2x} + (a-2)e^x - x$$

$$a > 0 \quad f'(x)$$

$$f'(x) \quad a$$

$$a > 0 \quad f(x) = 2ae^{2x} + (a-2)e^x - 1 = (2e^x + 1)(ae^x - 1)$$

$$f(x) = 0 \quad \therefore e^x = \frac{1}{a} \quad x = -\ln a$$

$$x \in (-\infty, -\ln a) \quad f'(x) < 0 \quad \therefore f(x) \quad (-\infty, -\ln a)$$

$$x \in (-\ln a, +\infty) \quad f'(x) > 0 \quad \therefore f(x) \quad (-\ln a, +\infty)$$

$$f(x) = 2ae^{2x} + (a-2)e^x - 1 = (2e^x + 1)(ae^x - 1)$$

$$a, 0 \quad f(x) < 0 \quad f(x) \quad R \quad f(x)$$

$$a > 0 \quad 1 \quad x = -\ln a \quad f(x)$$

$$f(-\ln a) = a \times \frac{1}{a} + (a-2) \times \frac{1}{a} + \ln a = 1 - \frac{1}{a} + \ln a < 0$$

$$u_a = 1 - \frac{1}{a} + \ln a \quad u_1 = 0$$

$$u_a = \frac{1}{a} + \frac{1}{a} > 0 \quad \therefore u(x) \quad (0, +\infty)$$

$$\therefore 0 < a < 1$$

$$x \rightarrow -\infty \quad f(x) \rightarrow +\infty \quad x \rightarrow +\infty \quad f(x) \rightarrow +\infty$$

$$\therefore f(x)$$

$$\therefore a \quad (0, 1)$$

10  $f(x) = (x-1)e^{2x} + ax^2 - ax$

1  $f(x)$

2  $f(x)$   $a$

1  $f(x) = e^{2x} + 2(x-1)e^{2x} + 2ax - a = (2x-1)(e^{2x} + a)$  1

1  $a > 0$   $e^{2x} + a > 0$   $f(x) > 0$   $x > \frac{1}{2}$   $f(x)$   $(\frac{1}{2}, +\infty)$

$f(x)$   $(-\infty, \frac{1}{2})$  2

2  $a < 0$   $f(x) = 0$   $x = \frac{1}{2}$   $x = \frac{\ln(-a)}{2}$

$\frac{\ln(-a)}{2} > \frac{1}{2}$   $a < -e$   $f(x) > 0$   $x < \frac{1}{2}$   $x > \frac{\ln(-a)}{2}$

$f(x)$   $(-\infty, \frac{1}{2})$   $(\frac{\ln(-a)}{2}, +\infty)$  3

$f(x)$   $(\frac{1}{2}, \frac{\ln(-a)}{2})$

$\frac{\ln(-a)}{2} = \frac{1}{2}$   $a = -e$

$x = \frac{1}{2}$   $x = 1, 0$   $e^{2x} + a, e^2 - e = 0$   $f(x) = 0$   $x > \frac{1}{2}$   $f(x) > 0$

$f(x)$   $(-\infty, +\infty)$  4

$\frac{\ln(-a)}{2} < \frac{1}{2}$   $-e < a < 0$   $f(x) > 0$   $x < \frac{\ln(-a)}{2}$   $x > \frac{1}{2}$

$f(x)$   $(-\infty, \frac{\ln(-a)}{2})$   $(\frac{1}{2}, +\infty)$   $f(x)$   $(\frac{\ln(-a)}{2}, \frac{1}{2})$  5

$a < -e$   $f(x)$   $(-\infty, \frac{1}{2})$   $(\frac{\ln(-a)}{2}, +\infty)$   $(\frac{1}{2}, \frac{\ln(-a)}{2})$

$a = -e$   $f(x)$   $(-\infty, +\infty)$



□  $-e < a < 0$  □  $f(x)$  □ □ □ □ □ □ □ □  $(-\infty, \frac{\ln(-a)}{2})$  □  $(\frac{1}{2}, +\infty)$  □ □ □ □ □ □ □ □  $(\frac{\ln(-a)}{2}, \frac{1}{2})$  □

□  $a > 0$  □  $f(x)$  □ □ □ □ □ □ □ □  $(\frac{1}{2}, +\infty)$  □ □ □ □ □ □ □ □  $(-\infty, \frac{1}{2})$  □ ..... □ 6 □

□ 2 □ □  $f(x) = (x-1)(e^{2x} + ax)$  □ □ □  $f(x)$  □ □ □ □  $x=1$  □ ..... □ 7 □

□  $f(x)$  □ □ □ □ □ □ □ □  $e^{2x} + ax = 0$  □ □ □ □ □ □ 1 □ □ □ □

□ 1 □ □  $g(x) = e^{2x} + ax$  □  $g'(x) = 2e^{2x} + a$  □

□ 1 □ □  $a > 0$  □ □  $g'(x) = 2e^{2x} + a > 0$  □ □ □ □ □ □ □  $g(x) = e^{2x} + ax$  □  $(-\infty, +\infty)$  □ □ □ □ □ □

□ □ □  $a > 0$  □  $g(0) = e^0 = 1 > 0$  □  $g(-\frac{1}{a}) = e^{-\frac{2}{a}} - 1 < 1 - 1 = 0$  □

□ □ □ □ □ □ □  $g(x)$  □  $(-\frac{1}{a}, 0)$  □ □ □ □ □ □ □

□ □  $g(x) = e^{2x} + ax$  □  $(-\infty, +\infty)$  □ □ □ □ □ □ □ □  $g(x)$  □ □ □ □ □ □ 1 □ □ □ □

□ □ □  $a > 0$  □ □ □ □ □ □ □ □ ..... □ 8 □

□ 2 □ □  $a = 0$  □ □  $g(x) = e^{2x}$  □ □ □ □ □ □ □ □

□ 3 □ □  $a < 0$  □ □ □  $g'(x) = 2e^{2x} + a > 0$  □ □ □  $x > \frac{1}{2} \ln(-\frac{a}{2})$  □

□  $g'(x) = 2e^{2x} + a < 0$  □ □ □  $x < \frac{1}{2} \ln(-\frac{a}{2})$  □

□ □  $g(x) = e^{2x} + ax$  □  $(-\infty, \frac{1}{2} \ln(-\frac{a}{2}))$  □ □ □ □ □ □ □ □  $(\frac{1}{2} \ln(-\frac{a}{2}), +\infty)$  □ □ □ □ □ □

□ □  $g(x) = e^{2x} + ax$  □  $x = \frac{1}{2} \ln(-\frac{a}{2})$  □ □ □ □ □ □ □ □ □ □ □ □ □ □

□ □ □ □  $g(x)_{\min} = g(\frac{1}{2} \ln(-\frac{a}{2})) = -\frac{a}{2} + \frac{a}{2} \ln(-\frac{a}{2})$  □ ..... □ 10 □

□ □ □  $g(x) = e^{2x} + ax$  □ □ □ □ □ □ □ □ 1 □ □ □ □

□ □ □  $x < 0$  □ □  $g(x) > 0$  □ □  $g(x)$  □  $(-\infty, \frac{1}{2} \ln(-\frac{a}{2}))$  □ □ □ □ □ □ □ □  $(\frac{1}{2} \ln(-\frac{a}{2}), +\infty)$  □ □ □ □ □ □

$$\square \quad g(x)_{\min} = g\left(\frac{1}{2} \ln\left(-\frac{a}{2}\right)\right) = -\frac{a}{2} + \frac{a}{2} \ln\left(-\frac{a}{2}\right) = 0 \quad \square \quad \begin{cases} g(1) = e^2 + a = 0 \\ g(x)_{\min} = g\left(\frac{1}{2} \ln\left(-\frac{a}{2}\right)\right) = -\frac{a}{2} + \frac{a}{2} \ln\left(-\frac{a}{2}\right) < 0 \end{cases}$$

$$\square \quad a = -2e \quad \square \quad a = -e^2 \quad \square \quad \dots \quad \square 11 \quad \square$$

$$\square \quad \square \quad a \quad \square \quad \square \quad \square \quad \{-2e, -e^2\} \cup (0, +\infty) \quad \square \quad \dots \quad \square 12 \quad \square$$

$$\square \quad 2 \quad \square \quad x=0 \quad \square \quad f(0) = -1 < 0 \quad \square \quad x=0 \quad \square \quad e^{2x} + ax = 0 \quad \square \quad \square \quad a = -\frac{e^{2x}}{x} \quad \square \quad \dots \quad \square 8 \quad \square$$

$$\square \quad g(x) = -\frac{e^{2x}}{x} \quad \square \quad \square \quad g'(x) = \frac{e^{2x}(1-2x)}{x^2} \quad \square$$

$$\square \quad g'(x) = \frac{e^{2x}(1-2x)}{x^2} > 0 \quad \square \quad x < \frac{1}{2} \quad \square \quad x \neq 0 \quad \square \quad g'(x) = \frac{e^{2x}(1-2x)}{x^2} < 0 \quad \square \quad x > \frac{1}{2} \quad \square$$

$$\square \quad g(x) \quad \square \quad (-\infty, 0) \quad \square \quad (0, \frac{1}{2}) \quad \square \quad \square \quad \square \quad \square \quad (\frac{1}{2}, +\infty) \quad \square \quad \square \quad \square \quad \dots \quad \square 9 \quad \square$$

$$\square \quad g(x) \quad \square \quad x = \frac{1}{2} \quad \square \quad \square \quad \square \quad \square \quad \square \quad g\left(\frac{1}{2}\right) = -2e \quad \square \quad \dots \quad \square 10 \quad \square$$

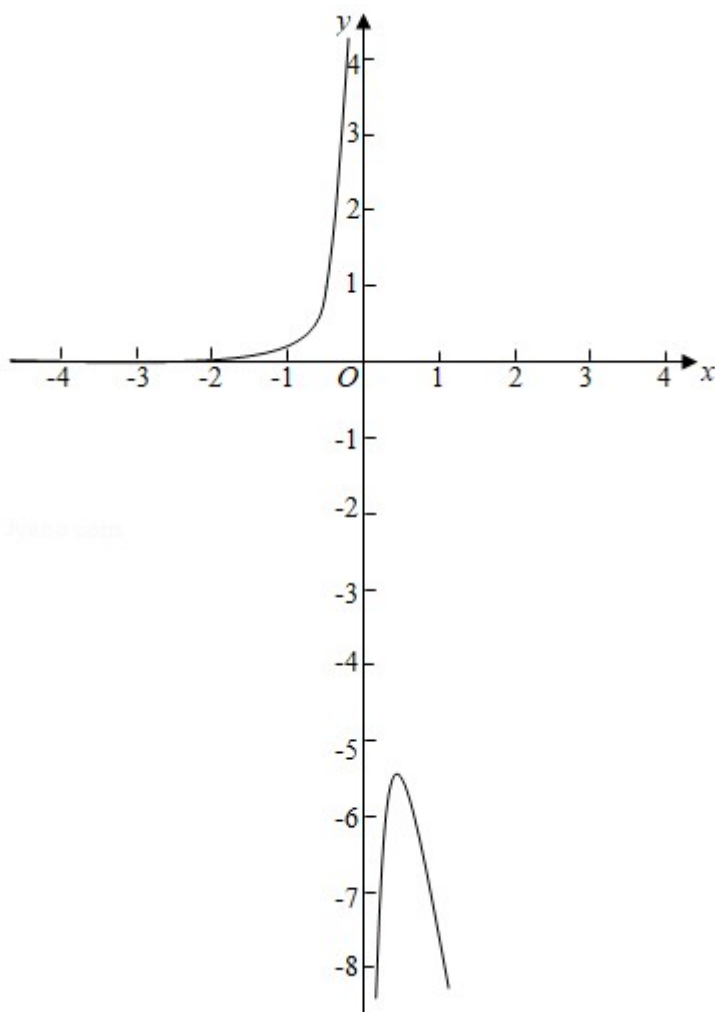
$$\square \quad g(x) = -\frac{e^{2x}}{x} \quad \square \quad \square \quad x < 0 \quad \square \quad g(x) > 0 \quad \square \quad x > 0 \quad \square \quad g(x)_{\min} = -2e \quad \square \quad \dots \quad \square 11 \quad \square$$

$$\square \quad e^{2x} + ax = 0 \quad \square \quad \square \quad \square \quad \square \quad \square \quad y = a \quad \square \quad g(x) \quad \square \quad \square \quad \square \quad \square$$

$$\square \quad \square \quad \square \quad a > 0 \quad \square \quad a = -2e \quad \square$$

$$\square \quad g \quad \square 1 \quad \square = -e \quad \square \quad \square \quad y = a \quad \square \quad g(x) \quad \square \quad \square \quad \square \quad \square \quad 1 \quad \square \quad \square \quad \square$$

$$\square \quad a \quad \square \quad \square \quad \square \quad \square \quad \square \quad \{-2e, -e^2\} \cup (0, +\infty) \quad \square \quad \dots \quad \square 12 \quad \square$$



11.  $f(x) = \frac{2x^2 - 1}{x} \cdot \ln(x) \quad (a \in \mathbb{R})$

$a > 0$   $f(x)$

$g(x) = f(x) - 2x$   $g(x)$   $a$

(I)  $f(x) = \frac{2x^2 - 1}{x} \cdot \ln(x) \quad (a \in \mathbb{R})$   $f(x) = \frac{2x^2 - ax + 1}{x^2}$   $x > 0$   $a > 0$

$y = 2x^2 - ax + 1$   $\Delta = a^2 - 8$

$a \in (0, 2\sqrt{2}]$   $\Delta \geq 0$   $f(x) \geq 0$   $f(x)$

$a \in (2\sqrt{2}, +\infty)$   $\Delta > 0$   $2x^2 - ax + 1 = 0$   $m, n$   $mn > 0$   $m + n > 0$   $m > 0$   $n > 0$

$$f(x) \text{ on } (0, n) \text{ and } (n, +\infty) \text{ and } (m, n)$$

$$(II) \quad g(x) = f(x) - 2x = -\frac{1}{x} - a \ln x \quad x > 0 \quad g(x) = \frac{1}{x^2} - \frac{a}{x} = \frac{1 - ax}{x^2}$$

$$a, 0 \quad g(x) > 0 \quad g(x) \text{ on } (0, +\infty) \quad g(x)$$

$$a > 0 \quad x \in (0, \frac{1}{a}) \quad g(x) \quad x \in (\frac{1}{a}, +\infty) \quad g(x)$$

$$\therefore g(x)_{\max} = g(\frac{1}{a}) = -a + a \ln a > 0 \quad a > e$$

$$a > e \quad 1 > \frac{1}{a} \quad g(1) = -1 < 0 \quad e^a > a, e^a < \frac{1}{a} \quad g(e^a) = -\frac{1}{e^a} - a \ln e^a = -\frac{1}{e^a} - a^2$$

$$h(x) = e^x - x^2 \quad h(x)$$

$$a > e \quad h(a) > h(e) = e^e - e^2 > 0 \quad e^e > e^2$$

$$g(e^a) = -\frac{1}{e^a} - a \ln e^a = -\frac{1}{e^a} - a^2 < 0$$

$$g(x) \text{ on } (e^a, \frac{1}{a}) \text{ and } (\frac{1}{a}, 1)$$

$$a > e$$

$$f(x) = \frac{1}{2}ax^2 - x - \ln x$$

$$a = 2$$

$$g(x) = f(x) + 2x^2 - \frac{3}{2}ax^2 + x + \ln x + h \quad h \in R \quad g(x)$$

$$f(x) \quad a$$

$$1 \text{ on } (0, +\infty) \quad a = 2 \quad f(x) = x^2 - x - \ln x$$

$$f(x) = 2x - 1 - \frac{1}{x} = \frac{(x-1)(2x+1)}{x}$$

$$0 < x < 1 \quad f(x) < 0 \quad x > 1 \quad f(x) > 0$$

$$f(x) \text{ on } (0, 1) \text{ and } (1, +\infty)$$



$$\lim_{x \rightarrow 0^+} m(x) = 0$$

$$x \in (1, +\infty) \implies m(x) < 0 \quad x \in (0, 1) \implies m(x) > 0$$

$$a > 2 \implies ax^2 = x^2 + 1 \implies x \in (0, 1) \implies f(x) > 0 \implies a > 2$$

$$a = 2 \implies f(x) = 0 \implies x = 1$$

$$0 < a < 2 \implies x_2 > 1 \implies f(x_2) < 0 \implies f\left(\frac{1}{e}\right) = \frac{a}{2e^2} - \frac{1}{e} + 1 > 0$$

$$x_0 > x_2 \implies f(x_0) > 0$$

$$\ln x, x-1 \implies \mu(x) = \ln x - x + 1$$

$$f(x) = \frac{ax^2}{2} - x - \ln x, \frac{ax^2}{2} - x - (x-1) = \frac{ax^2}{2} - 2x + 1 > \frac{ax^2}{2} - 2x = \frac{1}{2}ax\left(x - \frac{4}{a}\right)$$

$$x > \frac{4}{a} \implies \frac{1}{2}ax\left(x - \frac{4}{a}\right) > 0 \implies x_0 > \frac{4}{a} \implies f(x_0) > 0$$

$$0 < a < 2 \implies f(x)$$

$$a \in (0, 2)$$

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